NEET | IIT-JEE | FOUNDATION

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JEE MAINS-2019

10-04-2019 Online (Evening)

IMPORTANT INSTRUCTIONS

1. The test is of 3 hours duration.

2. This Test Paper consists of 90 questions. The maximum marks are 360.

3. There are three parts in the question paper A, B, C consisting of **Chemistry, Mathematics and Physics** having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for correct response.

4. Out of the four options given for each question, only one option is the correct answer.

5. For each incorrect response 1 mark i.e. ¼ (one-fourth) marks of the total marks allotted to the question will be deducted from the total score. No deduction from the total score, however, will be made if no response is indicated for an item in the Answer Box.

6. Candidates will be awarded marks as stated above in instruction No.3 for correct response of each question. One mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer box.

7. There is only one correct response for each question. Marked up more than one response in any question will be treated as wrong response and marked up for wrong response will be deducted accordingly as per instruction 6 above.

PART-A-CHEMISTRY

1. A hydrated solid X on heating initially gives a monohydrated compound Y. Y upon heating above 373K leads to an anhydrous white powder Z. X and Z, respectively, are:

(1) Baking soda and soda ash. (2) Washing soda and dead burnt plaster.

(3) Baking soda and dead burnt plaster. (4*) Washing soda and soda ash.

Sol. $\underset{\text{washin soda (X)}}{Na_2CO_3.10H_2O(s)} \xrightarrow{\Delta} \underset{\text{(Y)}}{Na_2CO_3.H_2O} \xrightarrow[\Delta]{T>373k} \underset{\text{(soda ash) (Z)}}{Na_2CO_3}$

2. Which of the following is NOT a correct method of the preparation of benzylamine from cyanobenzene?

(1*) (i) HCl/H_2O (ii) $NaBH_4$

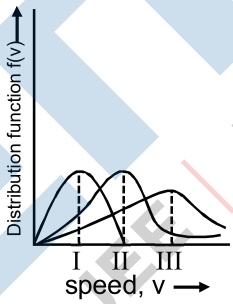
(2) (i) $SnCl_2$ + HCl(gas) (ii) $NaBH_4$

(3) (i) $LiAlH_4$ (ii) H_3O^+

(4) H_2/Ni

Sol. Benzylamine will not give cyanobenzene with HCl/H_2O & $NaBH_4$.

3. Points I, II and III in the following plot respectively correspond to (V_{mp} : most probable velocity)



(1) V_{mp} of H_2 (300K); V_{mp} of N_2(300K); V_{mp} of O_2(400K)

(2) V_{mp} of O_2 (400K); V_{mp} of N_2(300K); V_{mp} of H_2(300K)

(3) V_{mp} of N_2 (300K); V_{mp} of H_2(300K); V_{mp} of O_2(400K)

(4*) V_{mp} of N_2 (300K); V_{mp} of O_2(400K); V_{mp} of H_2(300K)

Sol. $V_{mp} = \sqrt{\frac{2RT}{M}} \Rightarrow V_{mp} \propto \sqrt{\frac{T}{M}}$

For N_2, O_2, H_2

$\sqrt{\frac{300}{28}} < \sqrt{\frac{400}{32}} < \sqrt{\frac{300}{2}}$

V_{mp} of N_2 (300K) < V_{mp} of O_2 (400k) < V_{mp} of H_2 (300K)

4. 1 g of non-volatile non-electrolyte solute is dissolved in 100g of two different solvents A and B whose ebullioscopic constants are in the ratio of 1 : 5. The ratio of the elevation in their boiling points $\frac{\Delta T_b(A)}{\Delta T_b(B)}$ is-

(1) 10 : 1 (2) 1 : 02 (3) 5 : 1 (4*) 1 : 5

Sol. $\Delta T_b = K_b \times m$

$\therefore \frac{\Delta T_{b(A)}}{\Delta T_{b(B)}} = \frac{\Delta K_{b(A)}}{\Delta K_{b(B)}}$ as $m_A = m_B$

$\therefore \frac{\Delta T_{b(A)}}{\Delta T_{b(B)}} = \frac{1}{5}$

5. The increasing order of nucleophilicity of the following nucleophiles is :

(a) $CH_3CO_2^-$ (b) H_2O (c) $CH_3SO_3^-$ (d) $\overset{\ominus}{O}H$

(1) (d) < (a) < (c) < (b) (2*) (b) < (c) < (a) < (d)

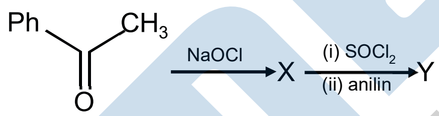
(3) (a) < (d) < (c) < (b) (4) (b) < (c) < (d) < (a)

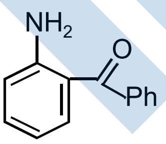
Sol. $\bar{O}H > CH_3-\underset{\underset{O}{\parallel}}{C}-O^- > CH_3-\overset{\overset{O}{\parallel}}{\underset{\underset{O}{\parallel}}{S}}-O^- > H_2O$

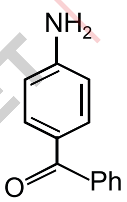
Charged ion | neutral system

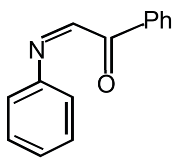
ione pair donating tendency on oxygen is reduced, nucleophilicity reduced b < c < a < d

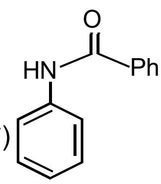
6. The major product 'Y' in the following reaction is:



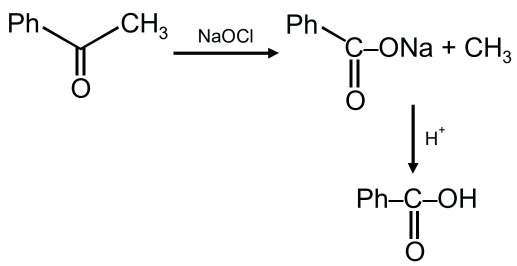
(1) 

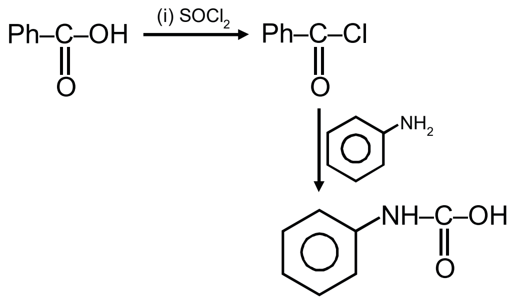
(2) 

(3) 

(4*) 

Sol.





7. Which of these factors does not govern the stability of a conformation in acyclic compounds ?

(1) Steric interactions
(2*) Angle strain
(3) Torsional strain
(4) Electrostatic forces of interaction

Sol. Angle strain govern stability in cyclic compound.

8. The ratio of the shortest wavelength of two spectral series of hydrogen spectrum is found to be about 9. The spectral series are:

(1*) Lyman and Paschen
(2) Brackett and Pfund
(3) Paschen and Pfund
(4) Balmer and Brackett

Sol.
$$\frac{\frac{1}{\lambda_2} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)z^2}{\frac{1}{\lambda_1} = R_H\left(\frac{1}{n_1^1} - \frac{1}{n_2^1}\right)z^2}$$

As for shortest wavelength both n_1 and n_2^{1} are ∞

$$\therefore \frac{\lambda_1}{\lambda_2} = \frac{9}{1} = \frac{n_1^1}{n_1^2}$$

Now if $n_1^1 = 3$ and n1 is 1 it will justify the statement hence Lyman and Paschen is correct.

9. The correct statements among (a) to (d) are:

(a) saline hydrides produce H_2 gas when reacted with H_2O.

(b) reaction of $LiAlH_4$ with BF_3 leads to B_2H_6.

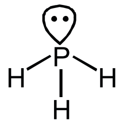
(c) PH_3 and CH_4 are electron-rich and electron precise hydrides, respectively.

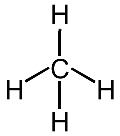
(d) HF and CH_4 are called as molecular hydrides.

(1) (c) and (d) only
(2*) (a), (b), (c) and (d)
(3) (a), (c) and (d) only
(4) (a), (b) and (c) only

Sol. (a) $\underset{\text{Ionic hydride}}{\overset{\oplus\ 1}{MH}} + HOH \longrightarrow MOH + H_2$

(b) $4BF_3 + 3LiAlH_4 \longrightarrow 2B_2H_6 + 3LiF + 3AlF_3$

(c)  → Phosphorous is electron rich hydride due to presence of lone pair

 → It is electron precise hydride.

(d) HF & CH_4 are molecular hydride due to they are covalent molecules.

10. The pH of a 0.02M NH_4Cl solution will be

[Given : $K_b(NH_4OH)=10^{-5}$ and log2=0.301]

(1) 4.35 (2) 2.65 (3) 4.65 (4*) 5.35

Sol. For the salt of strong acid and weak base

$$H^+ = \sqrt{\frac{K_w \times C}{K_b}}$$

$$\left[H^+\right] = \sqrt{\frac{10^{-14} \times 2 \times 10^{-2}}{10^{-5}}}$$

$$-\log\left[H^+\right] = 6 - \frac{1}{2}\log 20$$

$\therefore$ pH =5.35

11. The correct statement is :

(1) zincite is a carbonate ore

(2) sodium cyanide cannot be used in the metallurgy of silver

(3) zone refining process is used for the refining of titanium

(4*) aniline is a froth stabilizer

Sol. Fact base.

12. The INCORRECT statement is :

(1*) the gemstone, ruby, has Cr^{3+} ions occupying the octahedral sites of beryl.

(2) the spin-only magnetic moments of $[Fe(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$ are nearly similar.

(3) the color of $[CoCl(NH_3)_5]^{2+}$ is violet as it absorbs the yellow light.

(4) the spin-only magnetic moment of $[Ni(NH_3)_4(H_2O)_2]^{2+}$ is 2.83BM.

Sol. In gemstone, ruby has Cr^{3+} ion occupying the octahedral sites of aluminium oxide (Al_2O_3) normally occupied by Al^{3+} ion.

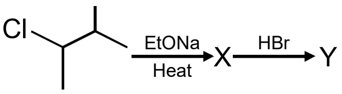
13. The crystal field stabilization energy (CFSE) of $[Fe(H_2O)_6]Cl_2$ and $K_2[NiCl_4]$, respectively, are :-

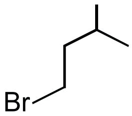
(1) $-0.6\Delta_0$ and $-0.8\Delta_t$ (2*) $-0.4\Delta_0$ and $-0.8\Delta_t$

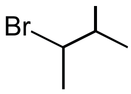
(3) $-2.4\Delta_0$ and $-1.2\Delta_t$ (4) $-0.4\Delta_0$ and $-1.2\Delta_t$

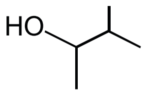
Sol. CFSE = $[-0.4n_{t2g} + 0.6\ n_{eg}]\ \Delta o$

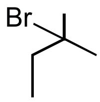
14. The major product 'Y' in the following reaction is:

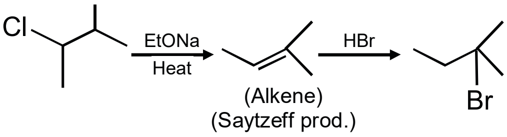


(1) 

(2) 

(3) 

(4*) 

Sol. 

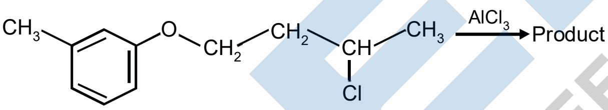
15. The correct order of the first ionization enthalpies is:

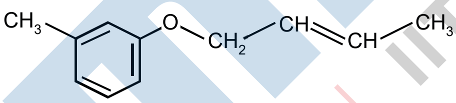
(1) Ti < Mn < Zn < Ni (2) Zn < Ni < Mn < Ti

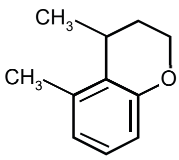
(3) Mn < Ti < Zn < Ni (4*) Ti < Mn < Ni < Zn

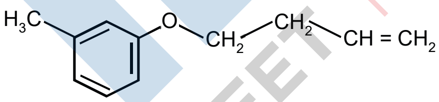
Sol. As Zn is fully filled and left to right in group IP increases.

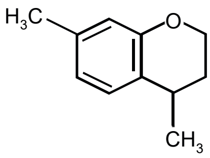
16. The major product obtained in the given reaction is :

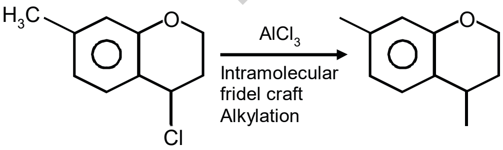


(1) 

(2) 

(3) 

(4*) 

Sol. 

17. Air pollution that occurs in sunlight is :

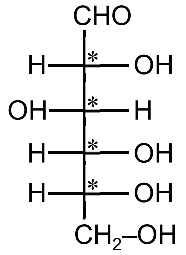
(1) acid rain (2*) oxidising smog (3) fog (4) reducing smog

Sol. Fact based.

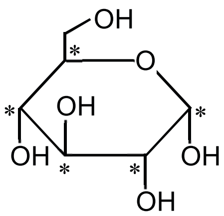
18. Number of stereo centers present in linear and cyclic structures of glucose are respectively :

(1) 4 & 4 (2*) 4 & 5 (3) 5 & 4 (4) 5 & 5

Sol.



D–Glucose
(Linear structure)
* ; - Stereocenter



α–Glucose
(cyclic structure)

19. For the reaction,

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$, $\Delta H = -57.2 kJ\ mol^{-1}$ and $K_C = 1.7 \times 10^{16}$.
Which of the following statement is INCORRECT?

(1*) The equilibrium constant is large suggestive of reaction going to completion and so no catalyst is required.

(2) The equilibrium constant decreases as the temperature increases.

(3) The equilibrium will shift in forward direction as the pressure increase.

(4) The addition of inert gas at constant volume will not affect the equilibrium constant.

Sol. In option (B)- Δn_g is –ve therefore increase in pressure will bring reaction in forward direction.

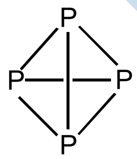
In option (C)- as the reaction is exothermic therefore increase in temperature will decrease the equilibrium constant.

In option (D)- Equilibrium constant changes only with temperature. Hence, option (B), (C) and (D) are correct therefore option (1) is incorrect choice.

20. The number of pentagons in C_{60} and trigons (triangles) in white phosphorus, respectively, are:

(1*) 12 and 4 (2) 20 and 3 (3) 20 and 4 (4) 12 and 3

Sol. Refer structure of C_{60} & P_4



21. The **correct** option among the following is :

(1*) Colloidal particles in lyophobic sols can be precipitated by electrophoresis.

(2) Brownian motion in colloidal solution is faster if the viscosity of the solution is very high.

(3) Colloidal medicines are more effective because they have small surface area.

(4) Addition of alum to water makes it unfit for drinking.

Sol. In electrophoresis precipitation occurs at the electrode which is oppositely charged therefore (A) is correct.

22. The correct match between Item-I and Item-II is:

Item-I	**Item-II**
(a) High density polythene	(I) Peroxide catalyst
(b) Polyacrylonitrile	(II) Condensation at high temperature & pressure
(c) Novolac	(III) Ziegler-Natta catalyst
(d) Nylon 6	(IV) Acid or base catalyst

(1) (a)→(III), (b) → (I), (c) → (II), (d) → (IV) (2) (a) → (II), (b) → (IV), (c) → (I), (d) → (III)

(3) (a) → (IV), (b) → (II), (c) → (I), (d) → (III) (4*) (a) → (III), (b) → (I), (c) → (IV), (d) → (II)

Sol.

(a) High density polythene	(III) Ziegler-Natta Catalyst
(b) Polyacrylonitrile	(I) Peroxide catalys t
(c) Novolac	(IV) Acid or base catalyst
(d) Nylon 6	(II) Condensation at high temperature & pressure

23. The highest possible oxidation states of uranium and plutonium, respectively, are :-

(1) 6 and 4 (2) 4 and 6 (3*) 6 and 7 (4) 7 and 6

Sol. The highest oxidation state of U and Pu is 6+ and 7+ respectively.

24. In chromatography, which of the following statements is INCORRECT for R_f?

(1) The value of R_f can not be more than one.

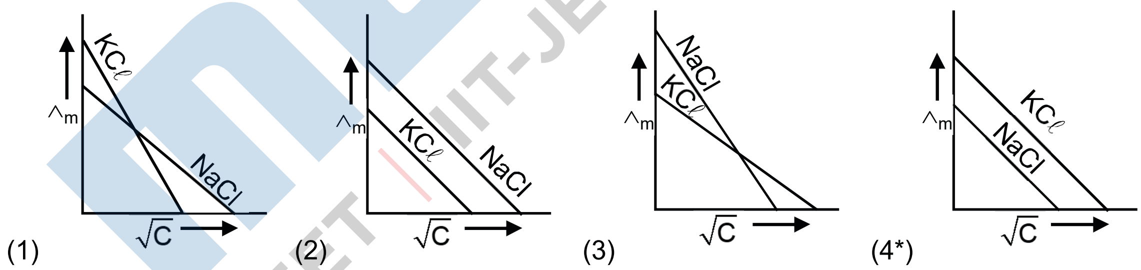
(2*) Higher R_f value means higher adsorption.

(3) R_f value is dependent on the mobile phase.

(4) R_f value depends on the type of chromatography.

Sol. R_f value can't measure the extent of adsorption.

25. Which one of the following graphs between molar conductivity ($\wedge_m$) versus $\sqrt{C}$ is correct?



Sol. Both NaCl and KCl are strong electrolytes and as Na^+(aq.) has less conductance than K^+ (aq.) due to more hydration therefore the graph of option (B) is correct.

26. The difference between ΔH and ΔU (ΔH–ΔU), when the combustion of one mole of heptane(l) is carried out at a temperature T, is equal to :

(1) 3RT (2*) –4RT (3) –3RT (4) 4RT

Sol. $C_7H_{16}(\ell) + 11O_2(g) \longrightarrow 7CO_2(g) + 8H_2O(\ell)$

$\Delta n_g = n_p - n_r = 7 - 11 = -4$

$\because \Delta H = \Delta U + \Delta n_g RT$

$\therefore \Delta H - \Delta U = -4RT$

27. For the reaction of H_2 with I_2, the rate constant is $2.5\times10^{-4} dm^3\ mol^{-1}\ s^{-1}$ at 327°C and 1.0 $dm^3\ mol^{-1}\ s^{-1}$ at 527°C. The activation energy for the reaction, in kJ mol^{-1} is: (R=8.314J $K^{-1}\ mol^{-1}$)

(1*) 166 (2) 150 (3) 59 (4) 72

Sol. $H_2(g) + I_2(g) \longrightarrow 2HI(g)$

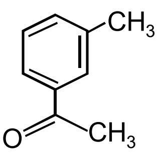
Apply Arrhenius equation

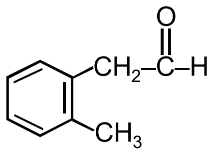
$$\log\frac{K_2}{K_1} = \frac{E_a}{2.303R}\left(\frac{1}{600}-\frac{1}{800}\right)$$

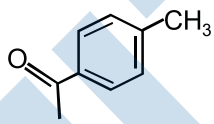
$$\log\frac{1}{2.5\times10^{-4}} = \frac{E_a}{2.303\times8.31}\left(\frac{200}{600\times800}\right)$$

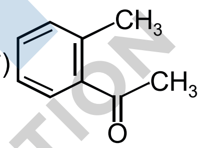
$\therefore E_a \approx 166$ kJ/mol

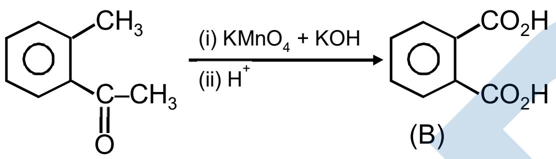
28. Compound A ($C_9H_{10}O$) shows positive iodoform test. Oxidation of A with $KMnO_4$/KOH gives acid B($C_8H_6O_4$). Anhydride of B is used for the preparation of phenolphthalein. Compound A is-

(1) 

(2) 

(3) 

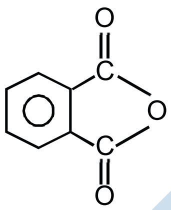
(4*) 

Sol. 

+ve iodoform test

(B)

($C_8H_6O_4$) Phthalic acid



is used for preparation of phenolphathalein indictor

Phthalic anhydride

29. The noble gas that does NOT occur in the atmosphere is:

(1) Kr (2) He (3) Ne (4*) Ra

Ans. BONUS

Sol. Fact based.

30. The minimum amount of $O_2(g)$ consumed per gram of reactant is for the reaction :

(**Given** atomic mass : Fe = 56, O = 16, Mg = 24, P = 31, C = 12, H = 1)

(1) $C_3H_8(g) + 5\ O_2(g) \to 3\ CO_2(g) + 4\ H_2O(l)$

(2*) $4\ Fe(s) + 3\ O_2(g) \to 2\ Fe_2O_3(s)$

(3) $2\ Mg(s) + O_2(g) \to 2\ MgO(s)$

(4) $P_4(s) + 5\ O_2(g) \to P_4O_{10}(s)$

Sol. 4 mol of Fe require 3 × 32 gram

$$\frac{1}{56}\text{ mol of Fe require } = \frac{3\times32}{4} = \frac{1}{56} = 0.428\text{ g}$$

PART-B-MATHEMATICS

31. The sum of the real roots of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$, is equal to

(1) 1 (2*) 0 (3) 6 (4) – 4

Sol. By expansion, we get

$-5x^3 + 30x - 30 + 5x = 0$

$\Rightarrow -5x^3 + 35x - 30 = 0$

$\Rightarrow x^3 - 7x + 6 = 0$, All roots area real

So, sum of roots = 0

32. If 5x + 9 = 0 is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is

(1) $\left(-\frac{5}{3}, 0\right)$ (2*) (– 5, 0) (3) $\left(\frac{5}{3}, 0\right)$ (4) (5, 0)

Sol. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

a = 3, b = 4 and $e = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$

corresponding focus will be (–ae, 0) i.e. (–5, 0).

33. If $\lim\limits_{x \to 1} \frac{x^2 - ax + b}{x - 1} = 5$, then a + b is equal to

(1*) – 7 (2) – 4 (3) 5 (4) 1

Sol. $\lim\limits_{x \to 1} \frac{x^2 - ax + b}{x - 1} = 5$

1 – a + b = 0 ....(i)

2 – a = 5 ....(ii)

$\Rightarrow$ a + b = –7

34. If z and ω are two complex numbers such that |zω| = 1 and arg(z) – arg(ω) = $\frac{\pi}{2}$, then

(1) $z\bar{\omega} = \frac{-1+i}{\sqrt{2}}$ (2*) $\bar{z}\omega = -i$ (3) $\bar{z}\omega = i$ (4) $z\bar{\omega} = \frac{1-i}{\sqrt{2}}$

Sol. $|z|.|w| = 1$ $z = re^{i\left(\theta + \frac{\pi}{2}\right)}$ and $w = \frac{1}{r}e^{i\theta}$

$\bar{z}.w = e^{-i\left(\theta + \frac{\pi}{2}\right)}.e^{i\theta} = e^{-i\left(\frac{\pi}{2}\right)} = -i$

$z.\bar{w} = e^{i\left(\theta+\frac{\pi}{2}\right)}.e^{i\theta} = e^{i\left(\frac{\pi}{2}\right)} = i$

35. If the line ax + y = c, touches both the curves $x^2 + y^2 = 1$ and $y^2 = 4\sqrt{2}x$, then | c | is equal to

(1) $\frac{1}{\sqrt{2}}$ (2) 2 (3) $\frac{1}{2}$ (4*) $\sqrt{2}$

Sol. Tangent to $y^2 = 4\sqrt{2}x$ is $y = mx + \frac{\sqrt{2}}{m}$ it is also tangent to $x^2 + y^2 = 1$

$\Rightarrow \left|\frac{\sqrt{2}/m}{\sqrt{1+m^2}}\right| = 1 \Rightarrow m \pm 1$

$\Rightarrow$ Tangent will be $y = x + \sqrt{2}$ or $y = -x - \sqrt{2}$ compare with y = –ax + C

$\Rightarrow$ a = ±1 and $C = x \pm \sqrt{2}$

36. Let a, b and c be in G.P. with common ratio r, where a ≠ 0 and $0 < r \le \frac{1}{2}$. If 3a, 7b and 15c are the first three terms of an A.P., then the 4^{th} term of this A.P. is

(1*) a (2) $\frac{2a}{3}$ (3) 5a (4) $\frac{7a}{3}$

Sol. b = ar

$c = ar^2$

3a,7b and 15c are in A.P.

$\Rightarrow$ 14b = 3a + 15c

$\Rightarrow 14(ar) = 3a + 15ar^2$

$\Rightarrow 14r = 3 + 15r^2$

$\Rightarrow 15r^2 - 14r + 3 = 0 \quad \Rightarrow (3r - 1)(5r - 3) = 0$

$r = \frac{1}{3}, \frac{3}{5}$

Only acceptable value is $r = \frac{1}{3}$, because $r \in \left(0, \frac{1}{2}\right]$

$\therefore$ c.d = 7b – 3a = 7ar – 3a $= \frac{7}{3}a - 3a = -\frac{2}{3}a$

$\therefore 4^{th}$ term $= 15c - \frac{2}{3}a = \frac{15}{9}a - \frac{2}{3}a = a$

37. Let $f(x) = \log_e(\sin x)$, $(0 < x < \pi)$ and $g(x) = \sin^{-1}(e^{-x})$, $(x \ge 0)$. If α is a positive real number such that a = (fog)'(α) and b = (fog)(α), then

(1) $a\alpha^2 + b\alpha - a = -2\alpha^2$ (2*) $a\alpha^2 - b\alpha - a = 1$

(3) $a\alpha^2 - b\alpha - a = 0$ (4) $a\alpha^2 + b\alpha + a = 0$

Sol. $fog(x) = (-x) \Rightarrow (fg(\alpha)) = -\alpha = b$

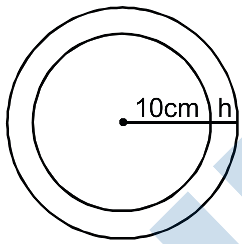
$(fg(x))' = -1 \Rightarrow (fg(\alpha))' = -1 = a$

38. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of the ice is 5 cm, then the rate at which the thickness (in cm/min) of the ice decreases, is

(1) $\frac{1}{9\pi}$ (2) $\frac{5}{6\pi}$ (3*) $\frac{1}{18\pi}$ (4) $\frac{1}{36\pi}$

Sol. $V = \frac{4}{3}\pi\left((10+h)^3 - 10^3\right)$

$\frac{dV}{dt} = 4\pi(10+h)^2\frac{dh}{dt}$



$-50 = 4\pi(10+5)^2\frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = -\frac{1}{18\pi}\frac{cm}{min}$

39. If $\int x^5 e^{-x^2}dx = g(x)e^{-x^2} + c$, where c is a constant of integration, then g(– 1) is equal to

(1) 1 (2) $-\frac{1}{2}$ (3*) $-\frac{5}{2}$ (4) – 1

Sol. Let $x^2 = t$ $2xdx = dt$

$\Rightarrow \frac{1}{2}\int t^2.e^{-t}dt = \frac{1}{2}\left[-t^2.e^{-t} + \int 2t.e^{-t},dt\right]$

$= \frac{1}{2}\left(-t^2.e^{-t}\right) + \left(-t.e^{-t} + \int 1.e^{-t}.dt\right)$

$= -\frac{t^2.e^{-t}}{2} - t.e^{-t} - e^{-t} = \left(-\frac{t^2}{2} - t - 1\right)e^{-t}$

$= \left(-\frac{x^4}{2} - X^2 - 1\right)e^{-x^2} + C$

for k = 0

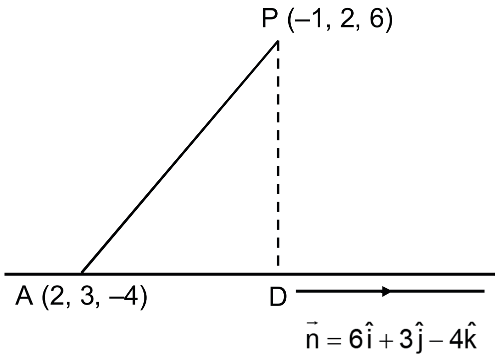
$g(-1) = -1 - 1 - \frac{1}{2} = -\frac{5}{2}$

40. The distance of the point having position vector $-\hat{i}+2\hat{j}+6\hat{k}$ from the straight line passing through the point (2, 3, – 4) and parallel to the vector, $6\hat{i}+3\hat{j}-4\hat{k}$ is

(1*) 7 (2) 6 (3) $2\sqrt{13}$ (4) $4\sqrt{3}$

Sol. $AD = \left|\frac{\overrightarrow{AP}.\vec{n}}{|\vec{n}|}\right| = \sqrt{61}$

$\Rightarrow PD = \sqrt{AP^2 - AD^2} = \sqrt{110 - 61} = 7$



41. The tangent and normal to the ellipse $3x^2 + 5y^2 = 32$ at the point P(2, 2) meet the x- axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is

(1) $\frac{14}{3}$ (2) $\frac{16}{3}$ (3) $\frac{34}{15}$ (4*) $\frac{68}{15}$

Sol. $3x^2 + 5y^2 = 32$

$\left.\frac{dy}{dx}\right|_{(2,2)} = -\frac{3}{5}$

Tangent : $y - 2 = -\frac{3}{5}(x - 2) \Rightarrow Q\left(\frac{16}{3}, 0\right)$

Normal : $y - 2 = -\frac{3}{5}(x - 2) \Rightarrow R\left(\frac{4}{5}, 0\right)$

Area is $= -\frac{1}{2}(QR) \times 2 = QR = \frac{68}{15}$

42. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is

(1*) 7 (2) 6 (3) 5 (4) 8

Sol. $1 - \left(\frac{1}{2}\right)^n > \frac{99}{100}$

$\Rightarrow \left(\frac{1}{2}\right)^n < \frac{1}{100}$

$\Rightarrow n = 7$

43. A perpendicular is drawn from a point on the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1}$ to the plane x + y + z =3 such that the foot of the perpendicular Q also lies on the plane x – y + z = 3. Then the co-ordinates of Q are

(1*) (2, 0, 1) (2) (1, 0, 2) (3) (– 1, 0, 4) (4) (4, 0, – 1)

Sol. Let point P on the line is $(2\lambda + 1, -\lambda - 1, \lambda)$ foot of perpendicular Q is given by

$$\frac{x-2\lambda-1}{1}=\frac{y+\lambda+1}{1}=\frac{z-\lambda}{1}=\frac{-(2\lambda-3)}{3}$$

$\because$ Q lies on x + y + z + 3 and x – y + z = 3

$\Rightarrow$ x + z = 3 and y = 0

$$y=0\Rightarrow\lambda+1=\frac{-2\lambda+3}{3}\Rightarrow\lambda=0$$

$\Rightarrow$ Q is (2, 0, 1)

44. If the plane 2x – y + 2z + 3 = 0 has the distances $\frac{1}{3}$ and $\frac{2}{3}$ units from the planes 4x – 2y +4z + λ = 0 and 2x – y + 2z + μ = 0, respectively, then the maximum value of $\lambda+\mu$ is equal to:

(1) 9 (2*) 13 (3) 5 (4) 15

Sol. 4x – 2y + 4z + 6 = 0

$$\frac{|\lambda-6|}{\sqrt{16+4+16}}=\left|\frac{\lambda-6}{6}\right|=\frac{1}{3}$$

$|\lambda-6|=2$

λ = 8,4

$$\frac{|\mu-3|}{\sqrt{4+4+1}}=\frac{2}{3}$$

$|\mu-3|=2$

μ = 5, 1

$\therefore$ Maximum value of $(\mu+\lambda)$ = 13.

45. If both the mean and the standard deviation of 50 observations x_1, x_2, ..., x_{50} are equal to 16, then the mean of $(x_1-4)^2$, $(x_2-4)^2$, ..., $(x_{50}-4)^2$ is:

(1) 525 (2) 480 (3*) 400 (4) 380

Sol. Mean $(\mu)=\frac{\sum x_i}{50}=16$

Standard deviation $(\sigma)=\sqrt{\frac{\sum_i^2}{50}-(\mu)^2}=16$

$$\Rightarrow(256)\times2=\frac{\sum x_i^2}{50}$$

$\Rightarrow$ New mean

$$=\frac{\sum(x_i-4)^2}{50}=\frac{\sum x_i^2+16\times50-8\sum x_i}{50}$$

= (256) × 2 + 16 – 8 ×16 = 400

46. The angles A, B and C of a triangle ABC are in A.P. and a : b = 1 : $\sqrt{3}$. If c = 4 cm, then the area (in sq. cm) of this triangle is:

(1) $4\sqrt{3}$ (2*) $2\sqrt{3}$ (3) $\frac{4}{\sqrt{3}}$ (4) $\frac{2}{\sqrt{3}}$

Sol. $\angle B = \frac{\pi}{3}$, by sine Rule

$\sin A = \frac{1}{2}$

$\Rightarrow A = 30°, a = 2,\ b = 2\sqrt{3},\ c = 4$

$\Delta = \frac{1}{2} \times 2\sqrt{3} \times 2 = 2\sqrt{3}$ sq. cm

47. If the tangent to the curve $y = \frac{x}{x^2 - 3}$, $x \in R, (x \neq \pm\sqrt{3})$, at a point $(\alpha, \beta) \neq (0, 0)$ on it is parallel to the line 2x + 6y – 11 = 0, then

(1) $|6\alpha + 2\beta| = 9$ (2) $|2\alpha + 6\beta| = 19$ (3*) $|6\alpha + 2\beta| = 19$ (4) $|2\alpha + 6\beta| = 11$

Sol. $\left.\frac{dy}{dx}\right|_{(\alpha,\beta)} = \frac{-\alpha^2 - 3}{(\alpha^2 - 3)^2}$

Given that :

$\frac{-\alpha^2 - 3}{(\alpha^2 - 3)^2} = -\frac{1}{3}$

$\Rightarrow \alpha = 0, \pm 3 \quad (\alpha \neq 0)$

$\Rightarrow \beta = \pm\frac{1}{2}. \quad (\beta \neq 0)$

$|6\alpha + 2\beta| = 19$

48. The area (in sq. units) of the region bounded by the curves y = 2^x and y = |x + 1|, in the first quadrant is

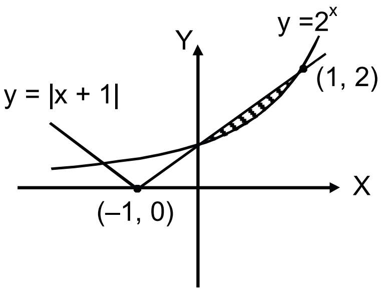
(1) $\frac{3}{2}$ (2) $\frac{1}{2}$ (3*) $\frac{3}{2} - \frac{1}{\ell og_e 2}$ (4) $\ell og_e 2 + \frac{3}{2}$

Sol. Required Area

$\int_0^1 ((x+1) - 2^x)\,dx$

$\left(\frac{x^2}{2} + x - \frac{2^x}{\ln 2}\right)_0^t$

$\left(\frac{1}{2} + 1 - \frac{2}{\ln 2}\right) - \left(0 + 0 - \frac{1}{\ln 2}\right)$



$=\frac{3}{2}-\frac{1}{\ln 2}$

49. The integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^{\frac{2}{3}} x \operatorname{cosec}^{\frac{4}{3}} x \, dx$ is equal to

(1) $3^{\frac{4}{3}}-3^{\frac{1}{3}}$ (2) $3^{\frac{5}{6}}-3^{\frac{2}{3}}$ (3*) $3^{\frac{7}{6}}-3^{\frac{5}{6}}$ (4) $3^{\frac{5}{3}}-3^{\frac{1}{3}}$

Sol. $I=\int \frac{1}{\cos^{2/3} x \sin^{1/3} x . \sin x} dx$

$=\int \frac{\tan^{2/3} x}{\tan^{4/3} x} . \sec^2 x . dx$

$=\int \frac{\sec^2 x}{\tan^{4/3} x} . dx$ {tan x = t, $\sec^2$ x dx = dt}

$=\int \frac{dt}{\tan^{4/3}}=\frac{t^{-1/3}}{-1/3}=-3\left(t^{-1/3}\right)$

$\Rightarrow$ 1 = -3 tan $(x)^{-1/3}$

$\Rightarrow I=\left.\frac{3}{(\tan x)^{1/3}}\right|_{\pi/6}^{\pi/3}=-3\left(\frac{1}{(\sqrt{3})^{1/3}}-(\sqrt{3})^{1/3}\right)$

$= 3^{7/6} - 3^{5/6}$

50. The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is

(1*) $y=\sqrt{1+2x},\ x\geq 0$ (2) $y=\sqrt{1+4x},\ x\geq 0$

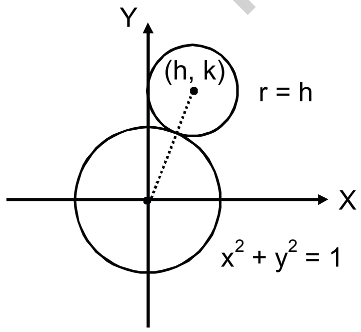
(3) $x=\sqrt{1+2y},\ y\geq 0$ (4) $x=\sqrt{1+4y},\ y\geq 0$

Sol. $\sqrt{h^2+k^2}=|h|+1$

$\Rightarrow x^2 + y^2 = x^2 + 1 - 2x$

$\Rightarrow y^2 = 1 + 2x$

$\Rightarrow y=\sqrt{1+2x}; x\geq 0$



51. Lines are drawn parallel to the line $4x - 3y + 2 = 0$, at a distance $\frac{3}{5}$ from the origin. Then which one of the following points lies on any of these lines?

(1) $\left(\frac{1}{4}, \frac{1}{3}\right)$ (2) $\left(\frac{1}{4}, -\frac{1}{3}\right)$ (3*) $\left(-\frac{1}{4}, \frac{2}{3}\right)$ (4) $\left(-\frac{1}{4}, -\frac{2}{3}\right)$

Sol. Required line is $4x - 3y + \lambda = 0$

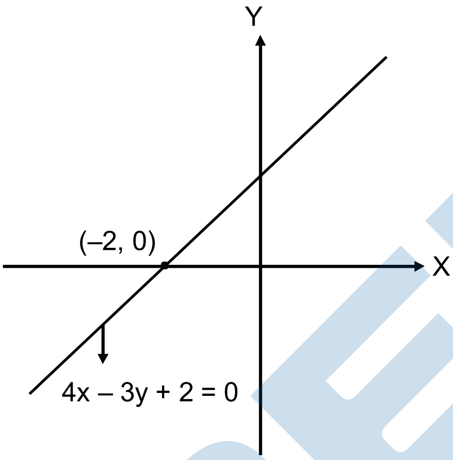
$\left|\frac{\lambda}{5}\right| = \frac{3}{5}$

$\Rightarrow \lambda = \pm 3$

So, required equation of line is

$4x - 3y + 3 = 0$ and $4x - 3y - 3 = 0$

(1) $4\left(-\frac{1}{4}\right) - 3\left(\frac{2}{3}\right) + 3 = 0$



52. The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is

(1) 2 (2*) 1 (3) 4 (4) 3

Sol. Let $2^x = t$

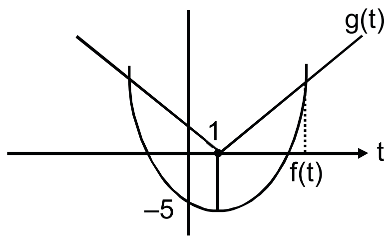
$5 + |t - 1| = t^2 - 2t$

$\Rightarrow |t - 1| = (t^2 - 2t - 5)$

g(t) f (t)

From the graph

So, number of real root is 1.



53. Let $y = y(x)$ be the solution of the differential equation, $\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that $y(0) = 1$. Then

(1*) $y'\left(\frac{\pi}{4}\right) - y'\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$ (2) $y\left(\frac{\pi}{4}\right) + y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{2} + 2$

(3) $y\left(\frac{\pi}{4}\right)-y\left(-\frac{\pi}{4}\right)=\sqrt{2}$ (4) $y'\left(\frac{\pi}{4}\right)+y'\left(\frac{-\pi}{4}\right)=-\sqrt{2}$

Sol. $\frac{dy}{dx}+y(\tan x)=2x+x^2\tan x$

$I.F.=e^{\pm\int \tan x\,dx}=e^{\ln.\sec x}=\sec x$

$\therefore$ y.sec x = $\int$ (2x + x^2 tan x)sec x.dx

= $\int$ 2x sec x dx + $\int$ x^2 (sec x.tan x)dx

y sec x = x^2 sec x + λ

$\Rightarrow$ y = x^2 + λ cos x

y (0) = 0 + λ = 1 $\Rightarrow \lambda = 1$

y = x^2 + cos x

$y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{16}+\frac{1}{\sqrt{2}}$

$y\left(-\frac{\pi}{4}\right)=\frac{\pi^2}{16}+\frac{1}{\sqrt{2}}$

y '(x) = 2x – sin x

$y'\left(-\frac{\pi}{4}\right)=\frac{\pi}{2}-\frac{1}{\sqrt{2}}$

$y'\left(\frac{-\pi}{4}\right)=\frac{-\pi}{2}+\frac{1}{\sqrt{2}}$

$y'\left(\frac{\pi}{4}\right)-y'\left(\frac{-\pi}{4}\right)=\pi+\sqrt{2}$

54. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2+\frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is

(1*) 38 (2) 58 (3) 35 (4) 23

Sol. $T_{r+1}=\sum_{r=0}^{n}{}^nC_r x^{2n-2r}.x^{-3r}$

2n – 5r = 1 $\Rightarrow$ 2n = 5r + 1 for r = 15, n = 38 smallest value of n is 38.

55. If $\cos^{-1}x-\cos^{-1}\frac{y}{2}=\alpha$, where $-1\le x\le 1, -2\le y\le 2,\ x\le\frac{y}{2}$, then for all x, y, $4x^2 - 4xy\cos\alpha + y^2$ is equal to

(1*) $4\sin^2\alpha$ (2) $2\sin^2\alpha$ (3) $4\sin^2\alpha - 2x^2y^2$ (4) $4\cos^2\alpha + 2x^2y^2$

Sol. $\cos^{-1}x-\cos^{-1}\frac{y}{2}=\alpha$

$\Rightarrow \cos\left(\cos^{-1}x-\cos^{-1}\frac{y}{2}\right)=\cos\alpha$

$$\Rightarrow x\times\frac{y}{2}+\sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}}=\cos\alpha$$

$$\Rightarrow\left(\cos\alpha-\frac{xy}{2}\right)^2=\left(1-x^2\right)\left(1-\frac{y^2}{4}\right)$$

$$x^2+\frac{y^2}{4}-xy\cos\alpha=1-\cos^2\alpha=\sin^2\alpha$$

56. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non - adjacent pillars, then the total number of beams is

(1*) 170 (2) 190 (3) 210 (4) 180

Sol. Total cases = number of diagonals in 20-sided polygon.

$= {}^{20}C_2 - 20 = 170$

57. The sum $1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+.....+\frac{1^3+2^3+3^3+...+15^3}{1+2+3+...15}-\frac{1}{2}(1+2+3+...+15)$ is equal to

(1) 1240 (2*) 620 (3) 1860 (4) 660

Sol. Sum $=\sum_{n=1}^{15}\frac{1^3+2^3+........n^3}{1+2+.......+n}-\frac{1}{2}.\frac{15.16}{2}$

$$=\sum_{n=1}^{15}\frac{n(n+1)}{2}-60$$

$$=\sum_{n=1}^{15}\frac{n(n+1)(n+2-(n-1))}{6}-60$$

$$=\frac{15.16.17}{6}-60=620$$

58. The negation of the Boolean expression $\sim S \vee (\sim r \wedge s)$ is equivalent to

(1) $\sim S \wedge \sim r$ (2*) $S \wedge r$ (3) $s \vee r$ (4) r

Sol. $\sim(\sim s \vee (\sim \wedge s))$

$s \wedge (r \vee \sim s)$

$(s \wedge r) \vee (s \wedge \sim s)$

$(s \wedge r) \vee (\phi)$

$(s \wedge r)$

59. Let λ be a real number for which the system of linear equations

$x + y + z = 6$

$4x + \lambda y - \lambda z = \lambda - 2$

$3x + 2y - 4z = -5$

has infinitely many solutions. Then λ is a root of the quadratic equation

(1) $\lambda^2+\lambda-6=0$ (2) $\lambda^2-3\lambda-4=0$ (3) $\lambda^2+3\lambda-4=0$ (4*) $\lambda^2-\lambda-6=0$

Sol. D = 0

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & \lambda \\ 3 & 2 & -4 \end{vmatrix} = 0 \Rightarrow = 3$$

60. Let a_1, a_2, a_3, ... be an A.P. with $a_6 = 2$. Then the common difference of this A.P., which maximises the product a_1 a_4 a_5, is

(1*) $\frac{8}{5}$ (2) $\frac{6}{5}$ (3) $\frac{2}{3}$ (4) $\frac{3}{2}$

Sol. Let a is first term and d is common difference then, a + 5d = 2 (given) ……(1)

f (d) = (2 – 5d)(2 – 2d)(2 – d)

PART-C-PHYSICS

61. In a Young's double slit experiment, the ratio of the slit's width is 4 : 1. The ratio of the intensity of maxima to minima, close to the central fringe on the screen, will be :

(1*) 9 : 1 (2) $\left(\sqrt{3}+1\right)^4 : 16$ (3) 4 : 1 (4) 25 : 9

Sol. $\frac{I_{Max}}{I_{Min}} = \frac{9}{1}$

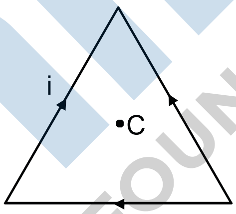
62. The magnitude of the magnetic field at the center of an equilateral triangular loop of side 1m which is carrying a current of 10 A is : [Take $\mu 0=4\pi\times10^{-7}$ NA^{-2}]

(1*) 18 μT (2) 3 μT (3) 9 μT (4) 1 μT

Sol. i = 10 A

ℓ = 1 m

$\mu_0 = 4\pi\times10^{-7}\frac{N}{A^2}$

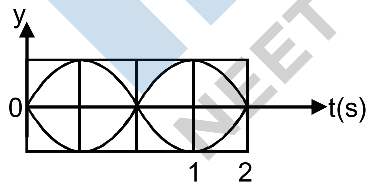


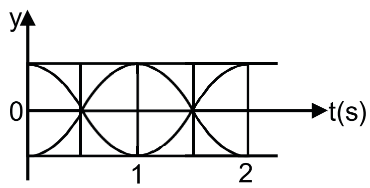
$B = \frac{\mu_0 i}{4\pi\frac{\sqrt{3}\ell}{2}}\times 3$ C

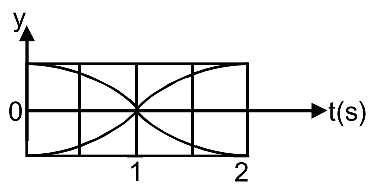
$= \frac{\mu_0 i\sqrt{3}}{2\pi\ell} = \frac{4\pi\times10^{-7}\times10\times\sqrt{3}}{2\pi\times1} = 20\sqrt{3}\times10^{-7}$

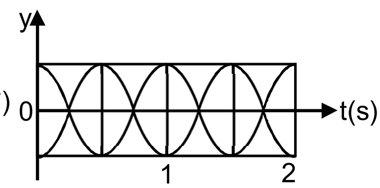
= 3 μT

63. The correct figure that shows, schematically, the wave pattern produced by superposition of two waves of frequencies 9 Hz and 11 Hz is :

(1) 

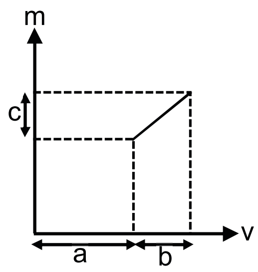
(2) 

(3) 

(4*) 

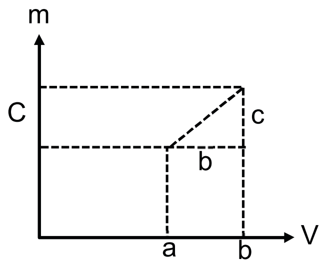
Sol. By looking into graph.

64. The graph shows how the magnification m produced by a thin lens varies with image distance v. What is the focal length of the lens used ?



(1) $\frac{b^2}{ac}$ (2) $\frac{b^2c}{a}$ (3*) $\frac{b}{c}$ (4) $\frac{a}{c}$

Sol. $f = \frac{b}{c}$



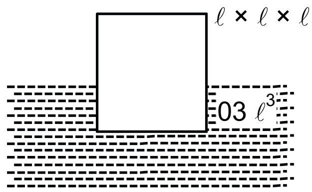
65. A cubical block of side 0.5 m floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water?

(Take density of water = 10^3 kg/m^3)

(1) 30.1 kg (2*) 87.5 kg (3) 46.3 kg (4) 65.4 kg

Sol. $0.3\ \ell^3\ \rho_\omega = \ell^3\ \rho$

$\rho = 300 \frac{kg}{m^3}$



$m + \ell^3\rho = \ell^3\ \rho_\omega$

$M = \ell^3(\rho_\omega - \rho) = (5)^3\ \{1000 - 300\} = 700 \times (5)^3$

= 87.5 kg

66. A 2 mW laser operates at a wavelength of 500 nm. The number of photons that will be emitted per second is : [Given Planck's constant h = 6.6 × 10^{-34} Js, speed of light c = 3.0 × 10^8 m/s]

(1*) 5 × 10^{15} (2) 1.5 × 10^{16} (3) 1 × 10^{16} (4) 2 × 10^{16}

Sol. $2 \times 10^{-3} = \frac{hc}{\lambda}\frac{dn}{dt}$

$\frac{dn}{dt} = \frac{2 \times 10^{-3}\lambda}{hc}$

$= \frac{2 \times 10^{-3} \times 500 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^{8}}$

$$= \frac{1000}{6.6 \times 3} \times 10^{14} = 5 \times 10^{15}$$

67. One mole of an ideal gas passes through a process where pressure and volume obey the relation $P = P_0 \left[1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$. Here P_0 and V_0 are constants. Calculate the change in the temperature of the gas if its volume changes from V_0 to $2V_0$.

(1) $\frac{1}{2}\frac{P_0V_0}{R}$ (2) $\frac{1}{4}\frac{P_0V_0}{R}$ (3*) $\frac{5}{4}\frac{P_0V_0}{R}$ (4) $\frac{3}{4}\frac{P_0V_0}{R}$

Sol. n = 1 mole

$$P = P_o\left\{1 - \frac{1}{2}\left(\frac{V_o}{V}\right)^2\right\}; \; PV = nRT = RT$$

$$P = \frac{RT}{V}$$

$$\frac{RT}{V} = P_o\left\{1 - \frac{V_o^2}{2V^2}\right\}$$

$$T = \frac{P_oV}{R}\left\{1 - \frac{V_o^2}{2V^2}\right\} = \frac{P_o}{R}\left\{V - \frac{V_o^2}{2V^2}\right\}$$

$$\Delta T = \frac{P_o}{R}\left\{(2V_o - V_o) - \frac{V_o^2}{2}\left(\frac{1}{2V_o} - \frac{1}{V_o}\right)\right\}$$

$$= \frac{P_o}{R}\left\{V_o - \frac{V_o^2}{2}\right\}$$

$$\Delta T = \frac{P_o}{R}\left\{(2V_o - V_o) - \frac{V_o^2}{2}\left(\frac{1}{2V_o} - \frac{1}{V_o}\right)\right\}$$

$$= \frac{P_o}{R}\left\{V_o - \frac{V_o^2(1-2)}{2 \times 2V_o}\right\}$$

$$= \frac{P_o}{R}\left\{V_o - \frac{V_o}{4}\right\} = \frac{3}{4}\frac{P_oV_o}{R}$$

68. A coil of self inductance 10 mH and resistance 0.1Ω is connected through a switch to a battery of internal resistance 0.9 Ω. After the switch is closed, the time taken for the current to attain 80% of the saturation value is : [Take ln 5 = 1.6]

(1*) 0.016 s (2) 0.103 s (3) 0.324 s (4) 0.002 s

Sol. $L = 10 \times 10^{-3}$ H, $r_1 = 0.1\ \Omega$

$i = \varepsilon \{1 - e^{-t/2}\}$

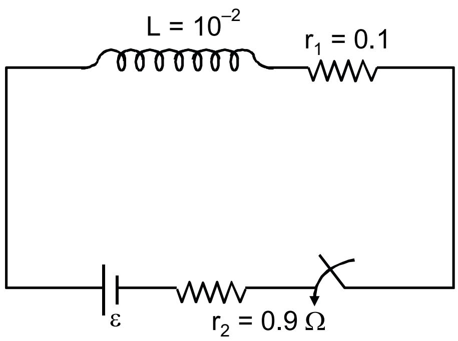
$i_{saturation} = \varepsilon$

80% $i_{saturation} = 0.8\ \varepsilon$

$0.8\ \varepsilon = \varepsilon \{1 - e^{-t/2}\}$

$0.8 = 1 - e^{-t/2}$; $e^{-t/2} = 0.2$

$e^{t/L} = 5$



$t = L \ln 5 = 10 \times 10^{-3} \times 1.6 = 16 \times 10^{-3}$

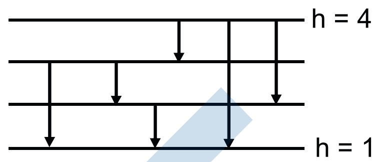
69. In Li^{++}, electron in first Bohr orbit is excited to a level by a radiation of wavelength . When the ion gets deexcited to the ground state in all possible ways (including intermediate emissions), a total of six spectral lines are observed. What is the value of λ ? (Given : $h = 6.63 \times 10^{-34}$ Js ; $c = 3 \times 10^{8}\ ms^{-1}$)

(1*) 10.8 nm (2) 9.4 nm (3) 12.3 nm (4) 11.4 nm

Sol. $\frac{hc}{\lambda} = 13.6\ eV(g)\left\{1 - \frac{1}{16}\right\}$

$\frac{1240\ eV}{\lambda} = \frac{15}{16} \times 9 \times 13.6\ eV$

$\lambda = \frac{1240 \times 16}{15 \times 9 \times 13.6} = 10.8\ nm$



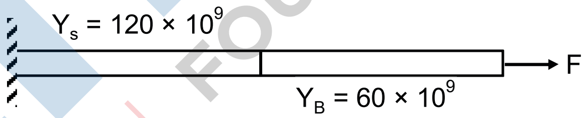
70. In an experiment, brass and steel wires of length 1m each with areas of cross section 1 mm^2 are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress required to produce a net elongation of 0.2 mm is : [Given, the Young's Modulus for steel and brass are respectively, $120 \times 10^{9}\ N/m^2$ and $60 \times 10^{9}\ N/m^2$]

(1) $1.8 \times 10^{6}\ N/m^2$ (2) $1.2 \times 10^{6}\ N/m^2$ (3) $0.2 \times 10^{6}\ N/m^2$ (4*) $4.0 \times 10^{6}\ N/m^2$

Sol. $\ell = 1\ M$

$A = 10^{-6}\ M^2$

$\text{Stress} = \frac{F}{A}$ $Y_s = 120 \times 10^{9}$



$\text{Stress} = \frac{\text{Stress}}{Y}$

$\Delta\ell = \frac{\ell \times F}{AY}$

$\Delta\ell_1 + \Delta\ell_2 = \frac{\ell_1 \times F}{AY_1} + \frac{\ell_2 \times F}{AY_2} = 0.2 \times 10^{-3}$

$\frac{F}{A} = \frac{0.2 \times 10^{-3}}{\frac{\ell}{Y_1} + \frac{\ell}{Y_2}}$

$= \frac{0.2 \times 10^{-3}}{\frac{1}{120 \times 10^{9}} + \frac{1}{60 \times 10^{9}}} = \frac{0.2 \times 10^{-3} \times 10^{9} \times 120}{1 + 2}$

$= \frac{0.2 \times 10^{6} \times 120}{3} = 8 \times 10^{6}$

Ans. None

71. A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet ?

[Given : Mass of planet = 8×10^{22} kg ; Radius of planet = 2×10^{6} m,

Gravitational constant G = 6.67×10^{-11} Nm^2/kg^2]

(1) 13 (2*) 11 (3) 9 (4) 17

Sol. $\frac{mV^2}{r} = \frac{GMm}{r^2}$

$V = \sqrt{\frac{GM}{r}}$

$n = \frac{VT}{2\pi r} = \sqrt{\frac{GM}{r}}\frac{T}{2\pi r}$

$= \left(\sqrt{\frac{GM}{r^3}}\right) \times \frac{T}{2\pi} = \sqrt{\frac{6.67 \times 10^{-11} \times 8 \times 10^{22}}{(202 \times 10^4)^3}} \times \frac{T}{2\pi}$

$= \frac{24 \times 3600}{2 \times 3.14}\sqrt{\frac{6.67 \times 8 \times 10^{11}}{(202)^3 \times 10^{12}}} = \frac{24 \times 3600}{2 \times 3.14 \times 1242.8} = \frac{24 \times 3600}{78.51} \approx 11$

72. When heat Q is supplied to a diatomic gas of rigid molecules, at constant volume its temperature increases by ΔT. The heat required to produce the same change in temperature, at a constant pressure is :

(1) $\frac{3}{2}Q$ (2) $\frac{5}{3}Q$ (3) $\frac{2}{3}Q$ (4*) $\frac{7}{5}Q$

Sol. $Q = C_V \Delta T$

$Q' = C_P \Delta T$

$Q' = \frac{C_P}{C_V}Q = \left(1 + \frac{2}{5}\right)Q = \frac{7}{5}Q$

73. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius 2R. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis. The ratio I_1/I_2 is given by :

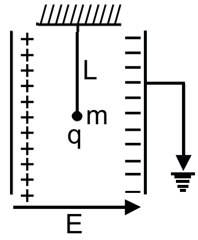
(1) 285 (2) 185 (3*) 140 (4) 65

Sol. $I_1 = \frac{\left(\frac{7M}{8}\right)(ZR)^2}{2} = \frac{7M \times 4R^2}{2 \times 8} = \frac{7MR^2}{4}$

$I_2 = \frac{2}{5}\frac{M}{8}\left(\frac{R}{2}\right)^2 = \frac{2M}{5 \times 8}\frac{R^2}{4} = \frac{MR^2}{80}$

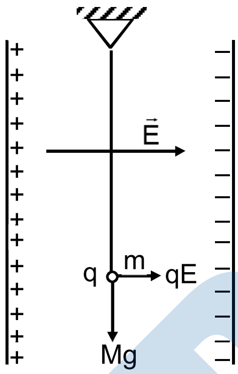
$$\frac{I_1}{I_2} = \frac{7MR^2 \times 80}{4MR^2} = 140$$

74. A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. The time period of the pendulum is given by:



(1) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2 - \dfrac{q^2E^2}{m^2}}}}$ (2*) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2 + \left(\dfrac{qE}{m}\right)^2}}}$ (3) $2\pi\sqrt{\dfrac{L}{\left(g - \dfrac{qE}{m}\right)}}$ (4) $2\pi\sqrt{\dfrac{L}{\left(g + \dfrac{qE}{m}\right)}}$

Sol. $T = 2\pi\sqrt{\dfrac{L}{\sqrt{g^2 + \dfrac{q^2E^2}{M^2}}}}$



75. In the formula X = 5YZ^2, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units ?

(1) $[M^{-2} L^{-2} T^6 A^3]$ (2) $[M^{-2} L^0 T^{-4} A^{-2}]$ (3) $[M^{-1} L^{-2} T^4 A^2]$ (4*) $[M^{-3} L^{-2} T^8 A^4]$

Sol. $X = 5YZ^2$

$$Y = \frac{X}{5Z^2} = M^{-3}L^{-2}T^8A^4$$

76. In free space, a particle A of charge 1 μC is held fixed at a point P. Another particle B of the same charge and mass 4 μg is kept at a distance of 1 mm from P. if B is released, then its velocity at a distance of 9 mm from P is : $\left[\text{Take } \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9 Nm^2C^{-2}\right]$

(1) 1.5×10^2 m/s (2*) 2.0×10^3 m/s (3) 1.0 m/s (4) 3.0×10^4 m/s

Sol. $q_A = 1\ \mu c$; $q_B = 1\ \mu c$, $m_B = 4 \times 10^{-9}$ kg, $r_{AB} = 10^{-3}$ m

$$\frac{1}{2}M_B V^2 = kq_A q_B \left\{\frac{1}{10^{-3}} - \frac{1}{9 \times 10^{-3}}\right\}$$

$$\frac{1}{2}4 \times 10^{-9} V^2 = 9 \times 10^9 \times 10^{-6} \times \frac{8}{9} \times 10^3$$

$$V^2 = \frac{8}{2} \times 10^9 = 4 \times 10^9$$

Ans. None

77. A bullet of mass 20 g has an initial speed of 1 ms^{-1}, just before it starts penetrating a mud wall of thickness 20 cm. if the wall offers a mean resistance of 2.5×10^{-2} N, the speed of the bullet after emerging from the other side of the wall is close to :

(1) 0.3 ms^{-1} (2) 0.4 ms^{-1} (3) 0.1 ms^{-1} (4*) 0.7 ms^{-1}

Sol. $$2.5 \times 10^{-2} \times 0.2 = \frac{1}{2} \times 20 \times 10^{-3}\{-V^2 + 1^2\}$$

$$5 \times 10^{-3} = 10 \times 10^{-3}(1 - V^2)$$

$$1 - V^2 = \frac{1}{2};\ V^2 = \frac{1}{2};\ V = \frac{1}{\sqrt{2}} = 0.7$$

78. The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod if it is to support a 400 N load without exceeding its elastic limit ?

(1) 0.90 mm (2*) 1.16 mm (3) 1.36 mm (4) 1.00 mm

Sol. $$\frac{400}{\frac{\pi}{4}d^2} = 379 \times 10^6$$

$$d^2 = \frac{4 \times 400 \times 10^{-6}}{\pi \times 379} = 0.336 \times 10^{-6} \times 4$$

$$d = 2\sqrt{0.336} \times 10^{-3} M \approx 1.16 \text{ mm}$$

79. Two radioactive substances A and B have decay constants 5λ and λ respectively. At t = 0, a sample has the same number of the two nuclei. The time taken for the ratio of the number of nuclei to become $\left(\frac{1}{e}\right)^2$ will be :

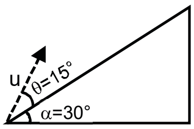
(1*) $\frac{1}{2\lambda}$ (2) $\frac{1}{4\lambda}$ (3) $\frac{1}{\lambda}$ (4) $\frac{2}{\lambda}$

Sol. $$\frac{1}{e^2} = e^{\lambda t - 5\lambda t}$$

$$t = \frac{1}{2\lambda}$$

70. A plane is inclined at an angle α=30° with respect to the horizontal. A particle is projected with a speed u = 2 ms^{-1} from the base of the plane, making an angle θ=15° with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to :

(Take g = 10 ms^{-2})



(1) 14 cm (2) 26 cm (3*) 20 cm (4) 18 cm

Sol. $T = \frac{2u\sin\theta}{g\cos\alpha}$

$R = u\cos\theta T - \frac{1}{2}g\sin\alpha T^2$

$= \frac{u\cos\theta 2u\sin\theta}{g\cos\alpha} - \frac{g\sin\alpha}{2}\frac{4u^2\sin^2\theta}{g^2\cos^2\alpha}$

$= \frac{u^2\sin^2\theta}{g\cos\alpha} - \frac{u^2\sin\alpha}{g\cos^2\alpha}\{1-\cos 2\theta\}$

$= \frac{4\times\frac{1}{2}}{10\times\frac{\sqrt{3}}{2}} - \frac{u^2\sin\alpha}{g\cos^2\alpha}\left\{1-\frac{\sqrt{3}}{2}\right\}$

$= \frac{4}{10\sqrt{3}} - \frac{8}{30}\left\{1-\frac{\sqrt{3}}{2}\right\}$

$= \frac{4}{5\sqrt{3}} - \frac{8}{30} = \frac{8\sqrt{3}-8}{30} = \frac{8(\sqrt{3}-1)}{30} = 20$ cm

81. A square loop is carrying a steady current I and the magnitude of its magnetic dipole moment is m. If this square loop is changed to a circular loop and it carries the same current, the magnitude of the magnetic dipole moment of circular loop will be :

(1) $\frac{m}{\pi}$ (2*) $\frac{4m}{\pi}$ (3) $\frac{3m}{\pi}$ (4) $\frac{2m}{\pi}$

Sol. $m = I\ell^2$ $2\pi r = 4\ell$

$m' = \frac{I4\ell^2}{\pi}$ $r = \frac{2\ell}{\pi}$

$\frac{m'}{m} = \frac{4}{\pi}$ $\pi r^2 = \frac{\pi 4\ell^2}{\pi^2} = \frac{4\ell^2}{\pi}$

$m' = \frac{4}{\pi}m$

82. Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1}. The cross sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of the stream, 0.15 m below the tap would be :

(Take g = 10 ms^{-2})

(1*) 5×10^{-5} m^2 (2) 5×10^{-4} m^2 (3) 1×10^{-5} m^2 (4) 2×10^{-5} m^2

Sol. $10^{-4} \times 1 = \sqrt{(1)^2 + 2 \times 10 \times 0.15} \times A$

$$A = \frac{10^{-4}}{2} = 5 \times 10^{-5}$$

83. Space between two concentric conducting spheres of radii a and b (b > a) is filled with a medium of resistivity . The resistance between the two spheres will be :

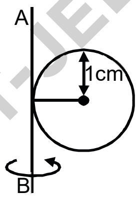
(1) $\frac{\rho}{2\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$ (2) $\frac{\rho}{4\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$

(3*) $\frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$ (4) $\frac{\rho}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$

Sol. $R = \int_a^b \frac{\rho dx}{4\pi x^2}$

$$= \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$

84. A metal coin of mass 5 g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5 s, is close to :



(1) 4.0×10^{-6} Nm (2*) 2.0×10^{-5} Nm (3) 7.9×10^{-6} Nm (4) 1.6×10^{-5} Nm

Sol. m = 5×10^{-3} kg, r = 10^{-2} m

$\omega = 25 \times 2\pi$ rad/5

= 50π rad/sec

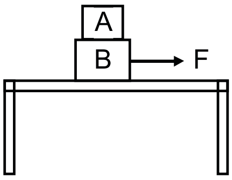
$$\omega = \frac{\tau}{I} t$$

$$\tau = \frac{I\omega}{t} = \frac{5mr^2}{4} \times \frac{\omega}{t}$$

$$= \frac{5 \times 5 \times 10^{-3} \times 10^{-4} \times 50\pi}{4 \times 5}$$

$$= \frac{25\pi}{4} \times 10^{-6} = 2 \times 10^{-5}$$

85. Two blocks A and B of masses m_A = 1 kg and m_B = 3 kg are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on B horizontally, so that the block A does not slide over the block B is : (Take g = 10 m/s²)

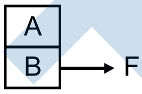


(1) 8 N (2) 12 N (3*) 16 N (4) 40 N

Sol. MA = 1 kg, MB = 3 kg

$\mu_{AB} = 0.2$

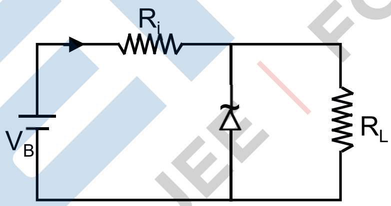
$\mu_B = 0.2$



$F_{max} = (M_A + M_B) \times 0.2 \times 10 + (M_A + M_B) \times 0.2 \times 10$

$= 4 \times 2 + 4 \times 2 = 16$

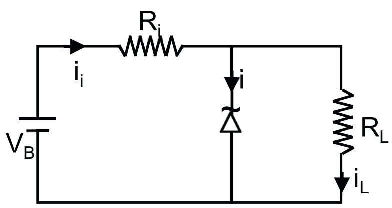
86. The figure represents a voltage regulator circuit using a Zener diode. The breakdown voltage of the Zener diode is 6V and the load resistance is R_L=4kΩ. The series resistance of the circuit is R_i=1kΩ. If the battery voltage V_B varies from 8V to 16V, what are the minimum and maximum values of the current through Zener diode?



(1) 0.5 mA; 6 mA (2*) 0.5 mA; 8.5 mA (3) 1.5 mA; 8.5 mA (4) 1 mA; 8.5 mA

Sol. $V_{breakdwon}$ = 6V, RL = 4kΩ, Ri = 1 kΩ

$i_L = \frac{6}{4} \times 10^{-3} = 1.5 \times 10^{-3} = 1.5$ mA



$i_i = 2 \times 10^{-3}$

$i = i_1 - i_L = 0.5$ mA – minimum current

$i_i = 10 \times 10^{-3} = 10$ mA

$i_{max} = 8.5$ mA

87. The time dependence of the position of a particle of mass m = 2 is given by $\vec{r}(t) = 2t\hat{i} - 3t^2\hat{j}$. Its angular momentum, with respect to the origin, at time t = 2 is :

(1) $48(\hat{i}+\hat{j})$ (2) $36\hat{k}$ (3) $-34(\hat{k}-\hat{i})$ (4*) $-48\hat{k}$

Sol. $\vec{v} = 2\hat{i} - 6 + \hat{j}$

At t = 2

$\vec{v} = 2\hat{i} - 12\hat{j}$

$\vec{P} = m\vec{v} = 4\hat{i} - 24\hat{j}$

At t = 2

$\vec{r} = 4\hat{i} - 24\hat{j}$

$$\vec{L} = \vec{r} \times \vec{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -12 & 0 \\ 4 & -24 & 0 \end{vmatrix}$$

$$= \{4(-24) + 4 \times 12\}\hat{k}$$

$$= (-96 + 48)\hat{k}$$

$$= (-)48\hat{k}$$

88. Light is incident normally on a completely absorbing surface with an energy flux of 25 Wcm^{-2}. If the surface has an area of 25 cm^2, the momentum transferred to the surface in 40 min time duration will be:

(1) 1.4×10^{-6} Ns (2) 6.3×10^{-4} Ns (3) 3.5×10^{-6} Ns (4*) 5.0×10^{-3} Ns

Sol. $I = 25\frac{W}{cm^2} = 25 \times 10^4 \ W/m^2$

P = 25 × 25 ; W = 625 W

$$\frac{hc}{\lambda}\frac{dn}{dt} = P$$

$$F = \frac{h}{\lambda}\frac{dn}{dt} = \frac{P}{C} = \frac{625}{3 \times 10^8}$$

$$\text{Momentum} = \frac{625 \times 40 \times 60}{3 \times 10^8} = 5 \times 10^{-3} \text{ Ns}$$

89. A submarine experiences a pressure of 5.05×10^6 Pa at a depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa., Then $d_2 - d_1$ is approximately (density of water = 10^3 kg/m^3 and acceleration due to gravity = 10 ms^{-2})

(1) 400 m (2) 500 m (3*) 300 m (4) 600 m

Sol. $P_1 = 5.05 \times 10^6$; $P_2 = 8.08 \times 10^6$

$P_2 - P_1 = \rho g(d_2 - d_1)$

$$d_2 - d_1 = \frac{3.03 \times 10^6}{10^3 \times 10} = 3.03 \times 10^2 = 303$$

90. A source of sound S is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him ? (Take velocity of sound in air is 350 m/s)

(1) 807 Hz (2) 857 Hz (3) 1143 Hz (4*) 750 Hz

Sol. $f_a = \frac{V}{V - V_s} f_o = 1000 \text{ Hz}$

s V = 50 m/s

$f_a' = \frac{V}{V + V_s} f_o$

$\frac{f_a'}{f_a} = \frac{V - V_s}{V + V_s} = \frac{350 - 50}{350 + 50} = \frac{300}{400} = \frac{3}{4}$

$f_a' = \frac{3}{4} \times 1000 = 750 \text{ Hz}$